SOME RELATIONSHIPS BETWEEN THE EMISSIVITIES OF VOLUMES AND THE COEFFICIENTS OF MUTUAL RADIATIVE HEAT TRANSFER BETWEEN THEM

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Inzhenerno-Fizicheskii Zhurnal, Vol. 12, No. 2, pp. 165–168, 1967

UDC 536.3

Formulas connecting the emissivities of contiguous volumes and the coefficients of mutual radiative heat transfer between them are obtained. The formulas can be used in calculations of radiative heat transfer by zonal methods.

DEFINITIONS

The emissivity of a radiating volume determines the radiation of the volume. There are local emissivities ε_k and an average emissivity ε over the volume. They are connected by the relationship

$$\varepsilon F = \sum \varepsilon_k F_k, \tag{1}$$

where F is the surface of the whole volume, and F_k are the parts of the surface F corresponding to the emissivities ε_k.

The amount of energy emitted by the surface of the volume is

$$Q = \varepsilon F E_0, \tag{2}$$

and that emitted by part of the surface is

$$Q_k = \varepsilon_k F_k E_0. \tag{3}$$

The coefficient of mutual radiative heat transfer between the volumes p and q in the case of gray emission [1] is

$$H_{V-V}(p, q) = \frac{1}{\pi} \int\limits_{V_p}\int\limits_{V_q} \frac{k_p k_q}{x^2} \exp(-k_c x)\, dV_p\, dV_q. \tag{4}$$

The amount of radiant energy of the volume p absorbed in volume q is

$$Q(p, q) = H_{V-V}(p, q) E_0. \tag{5}$$

BASIC RELATIONSHIPS

We consider the radiation of two volumes p and q contiguous over a common plane surface F_0 and having free surfaces F_α and F_β (Fig. 1). The radiation is gray.

In the choice of the shape of the volumes we introduce the restriction that a straight line passing through the surface of contact of the volumes cuts each free surface of the volumes only in one point.

In accordance with equality (1), we write

$$\varepsilon_p F_p = \varepsilon_{p\alpha} F_\alpha + \varepsilon_{p0} F_0,$$
$$\varepsilon_q F_q = \varepsilon_{q\beta} F_\beta + \varepsilon_{q0} F_0. \tag{6}$$

The radiation of the double volume is made up of the radiation of the volume p from the surface F_α, the radiation of the volume q from the surface F_β, and of part of the radiation of each of these volumes (that part not absorbed by the other volume) from their common surface.

The first component of the radiation is

$$Q_1 = \varepsilon_{p\alpha} F_\alpha E_0.$$

The second component is

$$Q_2 = \varepsilon_{q\beta} F_\beta E_0.$$

The third component is

$$Q_3 = [\varepsilon_{p0} F_0 - H_{V-V}(p, q)] E_0.$$

The fourth component is

$$Q_4 = [\varepsilon_{q0} F_0 - H_{V-V}(p, q)] E_0.$$

The total radiation of the double volume is

$$Q = \sum Q_i = [\varepsilon_{p\alpha} F_\alpha + \varepsilon_{q\beta} F_\beta + \varepsilon_{p0} F_0 + \varepsilon_{q0} F_0 - 2H_{V-V}(p, q)] E_0. \tag{7}$$

On the other hand, we can write

$$Q = \varepsilon_{p+q} F_{p+q} E_0. \tag{8}$$

We equate the right sides of expressions (7) and (8), after which we make a substitution from equations (6). We obtain

$$\varepsilon_{p+q} F_{p+q} = \varepsilon_p F_p + \varepsilon_q F_q - 2H_{V-V}(p, q). \tag{9}$$

If the radiation from the free surfaces of volumes p and q reaches the other volume the corresponding amounts of absorbed energy should be ignored in the calculation of the values of $H_{V-V}(p, q)$.

We now consider three adjacent volumes, which we denote by 1, 2, and 3 (Fig. 2).

We will consider the volume 1 + 2 + 3 as a double volume composed of volume 1 + 2 and volume 3. We construct Eq. (9) for it as

$$\varepsilon_{1+2+3} F_{1+2+3} = \varepsilon_{1+2} F_{1+2} + \varepsilon_3 F_3 - 2H_{V-V}(1+2, 3). \tag{10}$$

Applying Eq. (9) to the double volume 1 + 2, we write

$$\varepsilon_{1+2} F_{1+2} = \varepsilon_1 F_1 + \varepsilon_2 F_2 - 2H_{V-V}(1, 2). \tag{11}$$

According to the additivity rule, we can write

$$H_{V-V}(1+2, 3) = H_{V-V}(1, 3) + H_{V-V}(2, 3). \tag{12}$$

From Eqs. (10)–(12) we obtain

$$\varepsilon_{1+2+3} F_{1+2+3} = \varepsilon_1 F_1 + \varepsilon_2 F_2 + \varepsilon_3 F_3 - 2[H_{V-V}(1, 2) + H_{V-V}(1, 3) + H_{V-V}(2, 3)]. \tag{13}$$

In a similar way we can find the product εF for four or more contiguous volumes. For an arbitrary number of volumes we obtain

$$\varepsilon_\Sigma F_\Sigma = \sum_{i=1}^{i=n} \varepsilon_i F_i - 2 \sum_{i=1}^{i=n-1} \sum_{k=1}^{k=n-i} H_{V-V}(i,\ i+k). \quad (14)$$

COROLLARY

We write formula (9) for volumes 2 and 3

$$\varepsilon_{2+3} F_{2+3} = \varepsilon_2 F_2 + \varepsilon_3 F_3 - 2H_{V-V}(2,\ 3). \quad (15)$$

From formulas (13), (11), and (15) we can obtain

$$H_{V-V}(1,\ 3) = \frac{1}{2}(\varepsilon_{1+2} F_{1+2} + \varepsilon_{2+3} F_{2+3} - \varepsilon_{1+2+3} F_{1+2+3} - \varepsilon_2 F_2). \quad (16)$$

EXTENSION TO THE CASE OF SELECTIVE RADIATION

The obtained relationships were derived for gray radiation. They are also applicable to monochromatic radiation. We will show that for isothermic systems they are valid even when the emission is not gray.

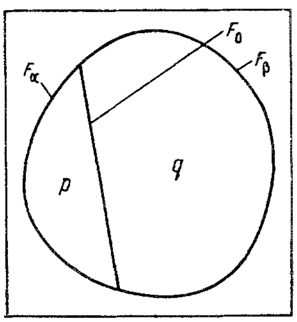

Fig. 1. Diagram of two contiguous radiating volumes.

The emissivity for nongray radiation [1] is

$$\varepsilon = \int_0^\infty \varepsilon_\lambda E_{0\lambda}\, d\lambda / \sigma_0 T^4. \quad (17)$$

The coefficient of mutual radiative heat transfer between the volumes is

$$H_{V-V}(p,\ q) = \int_0^\infty H_{V-V,\lambda}(p,\ q) E_{0\lambda}\, d\lambda / \sigma_0 T^4. \quad (18)$$

We write Eq. (9) for an elementary band of the spectrum of thickness $d\lambda$. We multiply it by $E_{0\lambda} d\lambda$ and integrate over the whole spectrum:

$$F_{p+q} \int_0^\infty \varepsilon_{p+q,\lambda} E_{0\lambda}\, d\lambda = F_p \int_0^\infty \varepsilon_{p\lambda} E_{0\lambda}\, d\lambda + F_q \int_0^\infty \varepsilon_{q\lambda} E_{0\lambda}\, d\lambda - 2 \int_0^\infty H_{V-V,\lambda}(p,\ q)\, d\lambda. \quad (19)$$

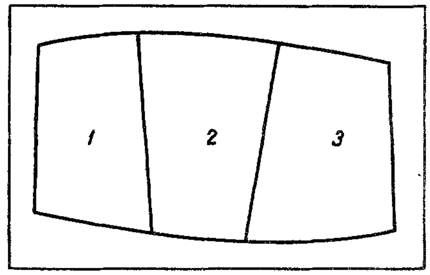

Fig. 2. Diagram of three adjacent radiating volumes.

Substituting in expression (19) for the integrals from Eqs. (17) and (18) and dividing by $\sigma_0 T^4$ we obtain

$$\varepsilon_{p+q} F_{p+q} = \varepsilon_p F_p + \varepsilon_q F_q - 2H_{V-V}(p,\ q). \quad (20)$$

This equation is the same as Eq. (9), but it is not restricted to gray or monochromatic emission and can be used in the case of selective emission also.

In a similar way we can show the validity of the other formulas for selective emission.

The obtained relationships can be used in calculations of radiative heat transfer between volumes.

NOTATION

$E = \sigma_0 T^4$ is the density of gray emission, ε_p, ε_q, ε_1,... are the emissivities of volumes p, q, 1..., ε_{p+q}, ε_{1+2}, ε_{1+2+3},... are the emissivities of volumes composed of parts p and q, 1 and 2, 1, 2, and 3, etc., ε_{p0}, $\varepsilon_{p\alpha}$, ε_{q0}, $\varepsilon_{q\beta}$,... are the local emissivities of volumes p and q on surfaces F_0, F_α, and F_β, $H_{V-V}(p, q)$, $H_{V-V}(1, 2)$, and $H_{V-V}(1+2, 3)$ are the coefficients of mutual radiative heat transfer for volumes p and q, volumes 1 and 2, a volume composed of parts 1 and 2 and volume 3, and so on, k is the absorption coefficient of the medium, k_p and k_q are the absorption coefficients for elementary volumes dV_p and dV_q, $k_c = \frac{1}{l}\int_0^l k\,dx$ is the mean absorption coefficient of the medium over path of the ray. The subscript λ denotes that the particular quantity relates to an infinitely small region of the spectrum defined by the wavelength λ.

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24 June 1966

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